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COMPUTATION OF SOMBOR INDICES OF OTIS(BISWAPPED) NETWORKS

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Abstract. In this paper, we derive analytical closed results for the first (a, b)-KA index, the Sombor index, the modified Sombor index, the first reduced (a, b)-KA index, the reduced Sombor index, the reduced modified Sombor index, the second reduced (a, b)-KA index and the mean Sombor index mSO_α for the OTIS biswapped networks by considering basis graphs as path, wheel graph, complete bipartite graph and $r-regular$ graphs. Network theory plays a significant role in electronic and electrical engineering, such as signal processing, networking, communication theory, and so on. A topological index (TI) is a real number associated with graph networks that correlates chemical networks with a variety of physical and chemical properties as well as chemical reactivity. The Optical Transpose Interconnection System (OTIS) network has recently received increased interest due to its potential uses in parallel and distributed systems.

1. Introduction

Cheminformatics is a new discipline of study that merges the scientific subjects of chemistry, information science, mathematics, and computer science. The quantitative structure-property relationship (QSPR) and quantitative structure-activity relationship (QSAR) are important aspects of cheminformatics that aid in the study of chemical compounds

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physicochemical properties. A topological index is a numeric value associated with a chemical compounds graph that describes its topology while being invariant under graph automorphism. In the study of structural chemistry, graph theory has a wide range of applications. Wiener's [17] investigation of paraffin boiling points was the first well-known application of a topological index in chemistry. Following that, numerous topological indices were introduced and explored in attempt to explain physicochemical features see [5, 22]. Also, numerous application of graph theory can be found in networking see [1, 6, 7, 13, 16].

Optical transpose interconnection system (OTIS) networks were designed to provide efficient connectivity for new optoelectronic computer architectures that took advantage of both optical and electronic technologies [14]. In OTIS networks, processors are organized into clusters. Electronic interconnects are utilised between processors within the same cluster, while optical links are utilised for intercluster communication. Numerous algorithms have been devised for routing, selection/sorting [10, 12, 18, 19], certain numerical computations [11], fourier transform [2], matrix multiplication [20] and image processing [21]. The structure of an interconnection network can be mathematically modeled by a graph. The vertices of this graph represent the processor nodes and the edges represent the links between the processors. The topology of a graph determines the way in which vertices are connected by edges. Certain aspects of a network may be easily identified from its topology. The greatest distance between any two nodes in the network determines the diameter. The degree of a node is determined by the number of links that connect to it. The network is said to be regular if this number is the same for all nodes. In this paper, G is considered to be a simple graph with V as vertex set and E as edge set and $|V| = n, |E| = m$. The *degree* d_v of a vertex $v \in V(G)$ is the number of edges incident to it in G.

Definition 1. [1] *For a base graph Ω, the biswapped interconnection network $Bsw(\Omega)$ is a graph with vertex set and edge set specified as:*
$V(Bsw(\Omega)) = \{\langle 0, p, g\rangle, \langle 1, p, g\rangle | p, g \in V(\Omega)\}$
$E(Bsw(\Omega)) = \{(\langle 0, p, g_1\rangle, \langle 0, p, g_2\rangle), (\langle 1, p, g_1\rangle, \langle 1, p, g_2\rangle) | (g_1, g_2) \in E(\Omega), p \in V(\Omega)\} \cup \{\langle 0, p, g\rangle, \langle 1, g, p\rangle | p, g \in V(\Omega)\}$.
The vertex and edge set cardinalities of biswapped network $Bsw(\Omega)$ are $2n^2$ and $2n|E(\Omega)| + n^2$ respectively, where n is the number of vertices in Ω. Figure 1 shows a biswapped network of star graph $K_{1,3}$ as the basis graph.

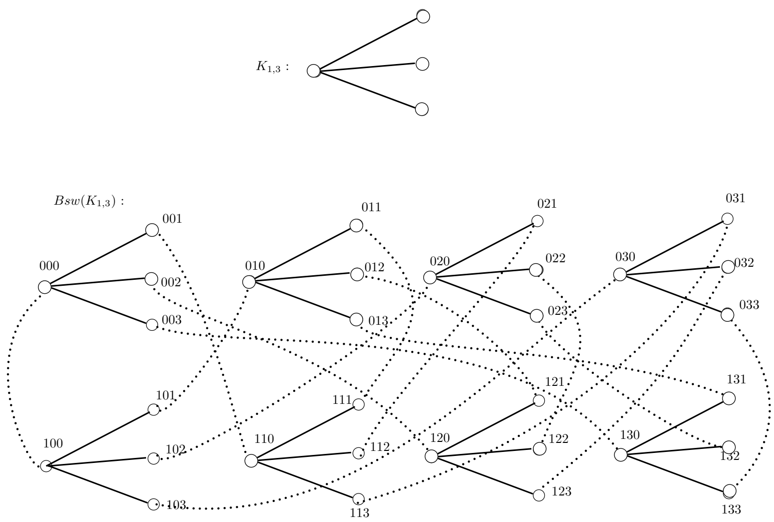


FIGURE 1. Biswapped network $K_{1,3}$

The sombor index was introduced by I. Gutman [8] to be described as

$$SO(G) = \sum_{uv \in E(G)} \sqrt{d_G(u)^2 + d_G(v)^2}.$$

The following indices are introduced by V. R. Kulli and I. Gutman [13]. The modified Sombor index of a graph G was [13] defined as

$$^{m}SO(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_G(u)^2 + d_G(v)^2}}.$$

The first (a, b)-KA index of a graph G was [13] defined as

$$KA^1_{a,b}(G) = \sum_{uv \in E(G)} [d_G(u)^a + d_G(v)^a]^b. \tag{1.1}$$

The reduced Sombor index of a graph G [8] was defined as

$$RSO(G) = \sum_{uv \in E(G)} \sqrt{(d_G(u) - 1)^2 + (d_G(v) - 1)^2}.$$

The reduced modified Sombor index of a graph G was [13] defined as

$$^{m}RSO(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{(d_G(u) - 1)^2 + (d_G(v) - 1)^2}}.$$

The first and second reduced (a,b)-KA indices of a graph G [13] were defined as

$$RKA^1_{a,b}(G) = \sum_{uv\in E(G)} \left[(d_G(u)-1)^a + (d_G(v)-1)^a\right]^b \quad (1.2)$$

$$RKA^2_{a,b}(G) = \sum_{uv\in E(G)} \left[(d_G(u)-1)^a \cdot (d_G(v)-1)^a\right]^b.$$

The mean Sombor index of a graph G was [15] defined as

$$mSO_\alpha(G) = \sum_{uv\in E(G)} \left(\frac{d_G(u)^\alpha + d_G(v)^\alpha}{2}\right)^{\frac{1}{\alpha}} \quad (1.3)$$

where $\alpha \in R\backslash 0$.

Table 1. [15] Expressions for the mean Sombor index $mSO_\alpha(G)$ for selected values of α

α	$mSO_\alpha(G)$	Index Equivalence
$-\infty$	$mSO_{\alpha\to-\infty}(G) = \sum_{uv\in E(G)} min(d_G(u), d_G(v))$	$SP_{\alpha\to-\infty}(G)$
-1	$mSO_{-1}(G) = \sum_{uv\in E(G)} \frac{2d_G(u)d_G(v)}{d_G(u)+d_G(v)}$	$2ISI(G)$
0	$mSO_{\alpha\to0}(G) = \sum_{uv\in E(G)} \sqrt{d_G(u)d_G(v)}$	$R^{-1}(G)$
$\frac{1}{2}$	$mSO_{\frac{1}{2}}(G) = \sum_{uv\in E(G)} \left(\frac{\sqrt{d_G(u)}+\sqrt{d_G(v)}}{2}\right)^2$	$2^{-2}KA^1_{\frac{1}{2},2}(G)$
1	$mSO_1(G) = \sum_{uv\in E(G)} \frac{d_G(u)+d_G(v)}{2}$	$2^{-1}M_1(G)$
2	$mSO_2(G) = \sum_{uv\in E(G)} \left(\frac{d_G(u)^2+d_G(v)^2}{2}\right)^{\frac{1}{2}}$	$2^{-\frac{1}{2}}SO(G)$
3	$mSO_3(G) = \sum_{uv\in E(G)} \left(\frac{d_G(u)^3+d_G(v)^3}{2}\right)^{\frac{1}{3}}$	$2^{-\frac{1}{3}}KA^1_{3,\frac{1}{3}}(G)$
∞	$mSO_{\alpha\to\infty}(G) = \sum_{uv\in E(G)} max(d_G(u), d_G(v))$	$SP_{\alpha\to\infty}(G)$

2. Methodology

There are three kinds of invariants:

1. Degree-based TIs
2. Distance-based TIs
3. Spectral-based TIs

In this paper, we concentrate on degree-based graph invariants. We partition the edge set of graph networks into classes depending on the degree of the end vertices and compute their cardinality to compute degree-based invariants. We compute our desired outcomes using this edge partition.

3. Computational results

In this section, we compute the first (a,b)-KA index, the Sombor index, the modified Sombor index, the first reduced (a,b)-KA index, the reduced Sombor index, the reduced modified Sombor index, the second reduced (a,b)-KA index and the mean Sombor index mSO_α for the OTIS biswapped networks by considering basis graphs as path, wheel graph, complete bipartite graph and $r-regular$ graphs.

3.1. Results for biswapped networks $Bsw(P_n)$

Let P_n be path on the n vertices and $Bsw(P_n)$ be the biswapped network with the basis network P_n [1]. The number of vertices and edges in $Bsw(P_n)$ are $2n^2$ and $3n^2 - 2n$ respectively. Figure 2 shows a OTIS biswapped network of path P_4 as the basis graph.

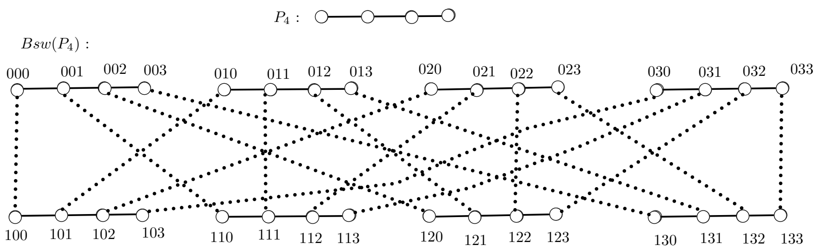


FIGURE 2. Biswapped network $Bsw(P_4)$

TABLE 2. The edge partition of the graph $Bsw(P_n)$

(d_u, d_v) where $uv \in E(Bsw(P_n))$	$(2,2)$	$(2,3)$	$(3,3)$
Number of edges	4	$8(n-1)$	$3n^2 - 10n + 4$

THEOREM 3.1. *The first (a,b)-KA index of biswapped network $Bsw(P_n)$ with basis network P_n is given by*
$KA^1_{a,b}(Bsw(P_n)) = 2^{b+ab+2} + 8(n-1)(2^a + 3^a)^b + (3n^2 - 10n + 4)2^b 3^{ab}$.

Proof. By definition and by using Table 2, we deduce

$$\begin{aligned} KA^1_{a,b}(Bsw(P_n)) &= \sum_{uv\in E(G)} [d_G(u)^a + d_G(v)^a]^b \\ &= 4(2^a+2^a)^b + 8(n-1)(2^a+3^a)^b \\ &\quad +(3n^2-10n+4)(3^a+3^a)^b \\ &= 4(2\times 2^a)^b + 8(n-1)(2^a+3^a)^b \\ &\quad +(3n^2-10n+4)(2\times 3^a)^b \\ &= 2^{b+ab+2} + 8(n-1)(2^a+3^a)^b \\ &\quad +(3n^2-10n+4)2^b 3^{ab}. \end{aligned}$$

□

From Theorem 3.1, we obtain the following results.

COROLLARY 3.2. *The Sombor index of $Bsw(P_n)$ is*

$$SO(Bsw(P_n)) = 8\sqrt{2} + 8\sqrt{13}(n-1) + 3\sqrt{2}(3n^2-10n+4).$$

COROLLARY 3.3. *The modified Sombor index of $Bsw(P_n)$ is*

$$^{m}SO(Bsw(P_n)) = \frac{3n^2-10(n-1)}{3\sqrt{2}} + \frac{8(n-1)}{\sqrt{13}}.$$

THEOREM 3.4. *The first reduced (a,b)-KA index of biswapped network $Bsw(P_n)$ with basis network P_n is given by*

$$RKA^1_{a,b}(Bsw(P_n)) = 2^{b+2}1^a + 8(n-1)(1^a+2^a)^b + (3n^2-10n+4)2^{b(a+1)}.$$

Proof. By definition and by using Table 2, we deduce

$$\begin{aligned} RKA^1_{a,b}(Bsw(P_n)) &= \sum_{uv\in E(G)} \left[(d_G(u)-1)^a + (d_G(v)-1)^a\right]^b \\ &= 4((2-1)^a + (2-1)^a)^b \\ &\quad +8(n-1)\big((2-1)^a + (3-1)^a\big)^b \\ &\quad +(3n^2-10n+4)(2^a+2^a)^b \\ &= 2^{b+2}1^a + 8(n-1)(1^a+2^a)^b \\ &\quad +(3n^2-10n+4)2^{b(a+1)}. \end{aligned}$$

□

From Theorem 3.4, we establish the following results.

Corollary 3.5. *The reduced Sombor index of $Bsw(P_n)$ is*

$$RSO(Bsw(P_n)) = 4\sqrt{2} + 8\sqrt{5}(n-1) + 2\sqrt{2}(3n^2 - 10n + 4).$$

Corollary 3.6. *The reduced modified Sombor index of $Bsw(P_n)$ is*

$$^{m}RSO(Bsw(P_n)) = \frac{10\sqrt{2} + 8\sqrt{5}(n-1)}{5} + \frac{\sqrt{2}(3n^2 - 10n + 4)}{4}.$$

Theorem 3.7. *The second reduced (a,b)-KA index of biswapped network $Bsw(P_n)$ with basis network P_n is given by*

$$RKA^2_{a,b}(Bsw(P_n)) = (3n^2 - 10n + 4)2^{2ab} + 2^{ab+3}1^{ab}(n-1) + 4(1)^{2ab}.$$

Proof. By definition and by using Table 2, we deduce

$$\begin{aligned} RKA^2_{a,b}(Bsw(P_n)) &= \sum_{uv \in E(G)} \left[(d_G(u)-1)^a \cdot (d_G(v)-1)^a\right]^b \\ &= 4((2-1)^a \cdot (2-1)^a)^b \\ &\quad +8(n-1)\big((2-1)^a \cdot (3-1)^a\big)^b \\ &\quad +(3n^2 - 10n + 4)(2^a \cdot 2^a)^b \\ &= (3n^2 - 10n + 4)2^{2ab} \\ &\quad +2^{ab+3}1^{ab}(n-1) + 4(1)^{2ab}. \end{aligned}$$

□

Theorem 3.8. *The mean Sombor index of biswapped network $Bsw(P_n)$ with basis network P_n is given by*

$$mSO_\alpha(Bsw(P_n)) = 9n^2 - 30n + 20 + 8(n-1)\left(\frac{2^\alpha + 3^\alpha}{2}\right)^{\frac{1}{\alpha}}.$$

Proof. By definition and by using Table 2, we deduce

$$\begin{aligned} mSO_\alpha(Bsw(P_n)) &= \sum_{uv \in E(G)} \left(\frac{d_G(u)^\alpha + d_G(v)^\alpha}{2}\right)^{\frac{1}{\alpha}} \\ &= 4\left(\frac{2^\alpha + 2^\alpha}{2}\right)^{\frac{1}{\alpha}} + 8(n-1)\left(\frac{2^\alpha + 3^\alpha}{2}\right)^{\frac{1}{\alpha}} \\ &\quad +(3n^2 - 10n + 4)\left(\frac{3^\alpha + 3^\alpha}{2}\right)^{\frac{1}{\alpha}} \\ &= 9n^2 - 30n + 20 + 8(n-1)\left(\frac{2^\alpha + 3^\alpha}{2}\right)^{\frac{1}{\alpha}}. \end{aligned}$$

□

3.2. Results for biswapped networks $Bsw(W_n)$

DEFINITION 2. [9] *For $n \geq 4$, the wheel graph W_n is defined to be the graph $K_1 + C_{n-1}$.*

Let W_n be wheel graph on the n vertices and $Bsw(W_n)$ [3] be the OTIS biswapped network with the basis network W_n. The number of vertices and edges in $Bsw(W_n)$ are $2n^2$ and $5n^2 - 4n$ respectively. Figure 3 shows a biswapped network of wheel W_5 as the basis graph.

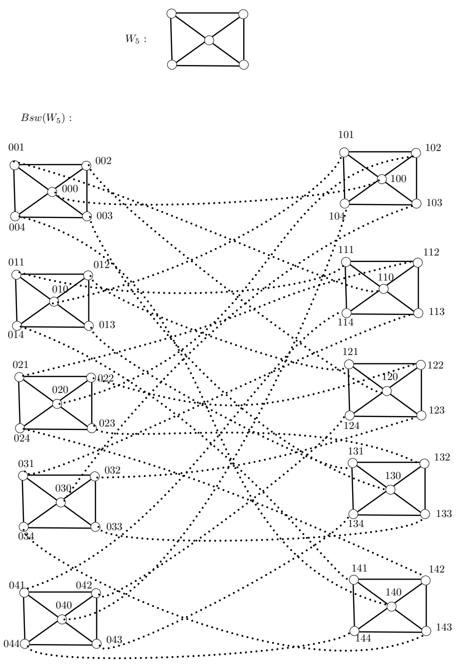


FIGURE 3. Biswapped network $Bsw(W_5)$

TABLE 3. The edge partition of the graph $Bsw(W_n)$

(d_u, d_v) where $uv \in E(Bsw(W_n))$	$(4,4)$	$(4,n)$	(n,n)
Number of edges	$(3n^2 - 4n + 1)$	$2(n^2 - 1)$	1

THEOREM 3.9. *The first (a,b)-KA index of biswapped network $Bsw(W_n)$ with basis network W_n is given by*

$KA^1_{a,b}(Bsw(W_n)) = 2^{b(1+2a)}(3n^2 - 4n + 1) + 2(n^2 - 1)(4^a + n^a)^b + 2^b n^{ab}.$

Proof. By definition and by using Table 3, we deduce

$$\begin{aligned} KA^1_{a,b}(Bsw(W_n)) &= \sum_{uv\in E(G)} [d_G(u)^a + d_G(v)^a]^b \\ &= (3n^2-4n+1)(4^a+4^a)^b + 2(n^2-1)(4^a+n^a)^b \\ &\quad +(n^a+n^a)^b \\ &= (3n^2-4n+1)(2\times 4^a)^b + 2(n^2-1)(4^a+n^a)^b \\ &\quad +(2\times n^a)^b \\ &= 2^{b(1+2a)}(3n^2-4n+1) + 2(n^2-1)(4^a+n^a)^b \\ &\quad +2^b n^{ab}. \end{aligned}$$

□

From Theorem 3.9, we obtain the following results.

Corollary 3.10. *The sombor index of $Bsw(W_n)$ is*

$$SO(Bsw(W_n)) = 4\sqrt{2}(3n^2-4n+1) + 2(n^2-1)\sqrt{n^2+16} + n\sqrt{2}.$$

Corollary 3.11. *The modified Sombor index of $Bsw(W_n)$ is*

$$^{m}SO(Bsw(W_n)) = \frac{3n^2-4n+1}{4\sqrt{2}} + \frac{2(n^2-1)}{\sqrt{n^2+16}} + \frac{1}{n\sqrt{2}}.$$

Theorem 3.12. *The first reduced (a,b)-KA index of biswapped network $Bsw(W_n)$ with basis network W_n is given by*

$$RKA^1_{a,b}(Bsw(W_n)) = 2^b3^{ab}(3n^2-4n+1)+2(n^2-1)(3^a+(n-1)^a)^b+2^b(n-1)^{ab}.$$

Proof. By definition and by using Table 3, we deduce

$$\begin{aligned} RKA^1_{a,b}(Bsw(W_n)) &= \sum_{uv\in E(G)} \left[(d_G(u)-1)^a + (d_G(v)-1)^a\right]^b \\ &= (3n^2-4n+1)(3^a+3^a)^b + 2(n^2-1)(3^a+(n-1)^a)^b \\ &\quad +((n-1)^a+(n-1)^a)^b \\ &= (3n^2-4n+1)(2\times 3^a)^b + 2(n^2-1)(3^a+(n-1)^a)^b \\ &\quad +(2\times(n-1)^a)^b \\ &= 2^b3^{ab}(3n^2-4n+1) + 2(n^2-1)(3^a+(n-1)^a)^b \\ &\quad +2^b(n-1)^{ab}. \end{aligned}$$

□

From Theorem 3.12, we establish the following results.

Corollary 3.13. *The reduced Sombor index of $Bsw(W_n)$ is*

$$RSO(Bsw(W_n)) = 3\sqrt{2}(3n^2-4n+1)+2\sqrt{n^2-2n+10}(n^2-1)+(n-1)\sqrt{2}.$$

Corollary 3.14. *The reduced modified Sombor index of $Bsw(W_n)$ is*

$$^{m}RSO(Bsw(W_n)) = \frac{(n-1)\sqrt{2}(3n^2-4n+1)+3\sqrt{2}}{6(n-1)} + \frac{2(n^2-1)}{\sqrt{n^2-2n+10}}.$$

Theorem 3.15. *The second reduced (a,b)-KA index of biswapped network $Bsw(W_n)$ with basis network W_n is given by*

$$RKA^2_{a,b}(Bsw(W_n)) = 3^{2ab}(3n^2-4n+1)+2(3^{ab})(n^2-1)(n-1)^{ab}+(n-1)^{2ab}.$$

Proof. By definition and by using Table 3, we deduce

$$\begin{aligned} RKA^2_{a,b}(Bsw(W_n)) &= \sum_{uv\in E(G)} \left[(d_G(u)-1)^a \cdot (d_G(v)-1)^a\right]^b \\ &= (3n^2-4n+1)(3^a\cdot 3^a)^b + 2(n^2-1)(3^a\cdot(n-1)^a)^b \\ &\quad +((n-1)^a\cdot(n-1)^a)^b \\ &= 3^{2ab}(3n^2-4n+1)+2(3^{ab})(n^2-1)(n-1)^{ab} \\ &\quad +(n-1)^{2ab}. \end{aligned}$$

□

Theorem 3.16. *The mean Sombor index of biswapped network $Bsw(W_n)$ with basis network W_n is given by*

$$mSO_\alpha(Bsw(W_n)) = 12n^2-15n+4+2(n^2-1)\left(\frac{4^\alpha+n^\alpha}{2}\right)^{\frac{1}{\alpha}}.$$

Proof. By definition and by using Table 3, we deduce

$$\begin{aligned} mSO_\alpha(Bsw(W_n)) &= \sum_{uv\in E(G)} \left(\frac{d_G(u)^\alpha+d_G(v)^\alpha}{2}\right)^{\frac{1}{\alpha}} \\ &= (3n^2-4n+1)\left(\frac{4^\alpha+4^\alpha}{2}\right)^{\frac{1}{\alpha}} \\ &\quad +2(n^2-1)\left(\frac{4^\alpha+n^\alpha}{2}\right)^{\frac{1}{\alpha}} + \left(\frac{n^\alpha+n^\alpha}{2}\right)^{\frac{1}{\alpha}} \\ &= 12n^2-15n+4+2(n^2-1)\left(\frac{4^\alpha+n^\alpha}{2}\right)^{\frac{1}{\alpha}}. \end{aligned}$$

□

3.3. Results for biswapped networks $Bsw(K_{r,s})$

Definition 3. [4] *A complete bipartite graph with partite sets V_1 and V_2, where $|V_1| = r$ and $|V_2| = s$, is then denoted by $K(r, s)$ or more commonly $K_{r,s}$. The graph $K_{1,s}$ is called a star.*

Let $K_{r,s}$ be complete bipartite graph on the $r+s$ vertices and $Bsw(K_{r,s})$ [3] be the biswapped network with the basis network $K_{r,s}$. The numbers of vertices and edges in $Bsw(K_{r,s})$ are $2n^2$ and $2nm + n^2$ respectively, where $n = r + s$ and $m = rs$. Figure 4 shows a biswapped network of complete bipartite $K_{2,3}$ as the basis graph.

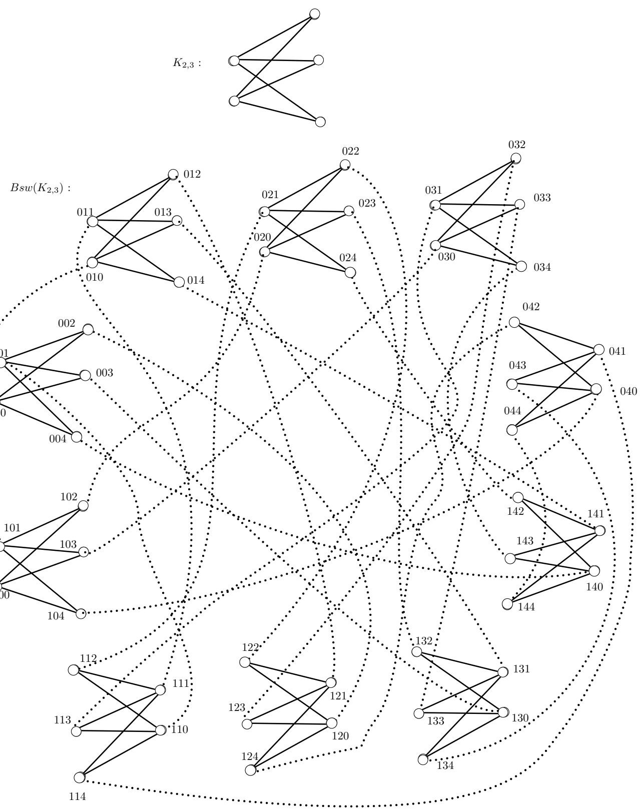


Figure 4. Biswapped network $Bsw(K_{2,3})$

Theorem 3.17. *The first (a, b)-KA index of biswapped network $Bsw(K_{r,s})$ with basis network $K_{r,s}$ is given by*

$$KA^1_{a,b}(Bsw(K_{r,s})) = 2rs(r+s+1)\big((r+1)^a+(s+1)^a\big)^b+r^2 2^b(s+1)^{ab}+s^2 2^b(r+1)^{ab}.$$

Table 4. The edge partition of the graph $Bsw(K_{r,s})$

(d_u, d_v) where $uv \in E(Bsw(K_{r,s}))$	$(r+1, s+1)$	$(s+1, s+1)$	$(r+1, r+1)$
Number of edges	$2rs(r+s+1)$	r^2	s^2

Proof. By definition and by using Table 4, we deduce

$$\begin{aligned} KA^1_{a,b}(Bsw(K_{r,s})) &= \sum_{uv\in E(G)} [d_G(u)^a + d_G(v)^a]^b \\ &= 2rs(r+s+1)\big((r+1)^a + (s+1)^a\big)^b \\ &\quad +r^2\big((s+1)^a + (s+1)^a\big)^b \\ &\quad +s^2\big((r+1)^a + (r+1)^a\big)^b \\ &= 2rs(r+s+1)\big((r+1)^a + (s+1)^a\big)^b \\ &\quad +r^2\big(2\times(s+1)^a\big)^b \\ &\quad +s^2\big(2\times(r+1)^a\big)^b \\ &= 2rs(r+s+1)\big((r+1)^a + (s+1)^a\big)^b \\ &\quad +r^2 2^b(s+1)^{ab} + s^2 2^b(r+1)^{ab}. \end{aligned}$$

□

From Theorem 3.17, we obtain the following results.

Corollary 3.18. *The sombor index of $Bsw(K_{r,s})$ is*

$$SO(Bsw(K_{r,s})) = 2rs(r+s+1)\sqrt{r^2+s^2+2(r+s+1)}+r^2\sqrt{2}(s+1)+s^2\sqrt{2}(r+1).$$

Corollary 3.19. *The modified Sombor index of $Bsw(K_{r,s})$ is*

$${}^mSO(Bsw(K_{r,s})) = \frac{2rs(r+s+1)}{\sqrt{r^2+s^2+2(r+s+1)}}+\frac{\sqrt{2}\Big(r^2(r+1)+s^2(s+1)\Big)}{2(r+1)(s+1)}.$$

Theorem 3.20. *The first reduced (a,b)-KA index of biswapped network $Bsw(K_{r,s})$ with basis network $K_{r,s}$ is given by*

$$RKA^1_{a,b}(Bsw(K_{r,s})) = 2rs(r+s+1)(r^a+s^a)^b + 2^b\big(r^2s^{ab}+s^2r^{ab}\big).$$

Proof. By definition and by using Table 4, we deduce

$$\begin{aligned} RKA^1_{a,b}(Bsw(K_{r,s})) &= \sum_{uv\in E(G)} \big[(d_G(u)-1)^a + (d_G(v)-1)^a\big]^b \\ &= 2rs(r+s+1)(r^a+s^a)^b + r^2\big(s^a+s^a\big)^b \\ &\quad + s^2\big(r^a+r^a\big)^b \\ &= 2rs(r+s+1)(r^a+s^a)^b + r^2\big(2\times s^a\big)^b \\ &\quad + s^2\big(2\times r^a\big)^b \\ &= 2rs(r+s+1)(r^a+s^a)^b + 2^b\big(r^2s^{ab}+s^2r^{ab}\big). \end{aligned}$$

□

From Theorem 3.20, we establish the following results.

COROLLARY 3.21. *The reduced Sombor index of $Bsw(K_{r,s})$ is*

$$RSO(Bsw(K_{r,s})) = 2rs(r+s+1)\sqrt{r^2+s^2} + rs\sqrt{2}\big(r+s\big).$$

COROLLARY 3.22. *The reduced modified Sombor index of $Bsw(K_{r,s})$ is*

$$^mRSO(Bsw(K_{r,s})) = \frac{2rs(r+s+1)}{\sqrt{r^2+s^2}} + \frac{\sqrt{2}(r^3+s^3)}{2rs}.$$

THEOREM 3.23. *The second reduced (a,b)-KA index of biswapped network $Bsw(K_{r,s})$ with basis network $K_{r,s}$ is given by*

$$RKA^2_{a,b}(Bsw(K_{r,s})) = 2rs(r+s+1)(r^as^a)^b + r^2s^{2ab} + s^2r^{2ab}.$$

Proof. By definition and by using Table 4, we deduce

$$\begin{aligned} RKA^2_{a,b}(Bsw(K_{r,s})) &= \sum_{uv\in E(G)} \big[(d_G(u)-1)^a \cdot (d_G(v)-1)^a\big]^b \\ &= 2rs(r+s+1)(r^a\cdot s^a)^b + r^2\big(s^a\cdot s^a\big)^b + s^2\big(r^a\cdot r^a\big)^b \\ &= 2rs(r+s+1)(r^as^a)^b + r^2s^{2ab} + s^2r^{2ab}. \end{aligned}$$

□

THEOREM 3.24. *The mean Sombor index of biswapped network $Bsw(K_{r,s})$ with basis network $K_{r,s}$ is given by*

$$mSO_\alpha(Bsw(K_{r,s})) = r^2(s+1)+s^2(r+1)+2rs(r+s+1)\left(\frac{(r+1)^\alpha+(s+1)^\alpha}{2}\right)^{\frac{1}{\alpha}}.$$

Proof. By definition and by using Table 4, we deduce

$$\begin{aligned} mSO_\alpha(Bsw(K_{r,s})) &= \sum_{uv\in E(G)} \left(\frac{d_G(u)^\alpha + d_G(v)^\alpha}{2}\right)^{\frac{1}{\alpha}} \\ &= 2rs(r+s+1)\left(\frac{(r+1)^\alpha + (s+1)^\alpha}{2}\right)^{\frac{1}{\alpha}} \\ &\quad +r^2\left(\frac{(s+1)^\alpha + (s+1)^\alpha}{2}\right)^{\frac{1}{\alpha}} \\ &\quad +s^2\left(\frac{(r+1)^\alpha + (r+1)^\alpha}{2}\right)^{\frac{1}{\alpha}} \\ &= 2rs(r+s+1)\left(\frac{(r+1)^\alpha + (s+1)^\alpha}{2}\right)^{\frac{1}{\alpha}} \\ &\quad +r^2(s+1) + s^2(r+1). \end{aligned}$$

□

Note: By putting r=1 in Theorems 3.17, 3.20, 3.23, 3.24 and Corollaries 3.18, 3.19, 3.21, 3.22, we get the first (a,b)-KA index, the first reduced (a,b)-KA index, the second reduced (a,b)-KA index, the mean Sombor index, the sombor index, the modified sombor index, the reduced Sombor index and the reduced modified Sombor index of biswapped network $Bsw(K_{1,s})$ with basis network $K_{1,s}$ (star graph) respectively.

3.4. Results for biswapped networks $Bsw(G_r)$

Definition 4. [4] *A graph G is regular of degree r if $deg(v) = r$ for each vertex v of G. Such graphs are called $r - regular$.*

Let G_r be $r - regular$ graph on the n vertices and $Bsw(G_r)$ [3] be the biswapped network with the basis network G_r. The numbers of vertices and edges in $Bsw(G_r)$ are $2n^2$ and $n^2(r+1)$ respectively. Figures 5, 6 and 7 show a biswapped network of cycle graph C_4, complete graph K_4 and n-cube graph Q_3 as the basis graph.

Theorem 3.25. *If $Bsw(G_r)$ is a biswapped network with basis network $r - regular$ graph G_r with order n, then*

$$mSO_\alpha(Bsw(G_r)) = n^2(r+1)^2.$$

Proof. The numbers of vertices and edges in $Bsw(G_r)$ are $2n^2$ and $n^2(r+1)$ respectively. By expression in (1.3) we get the desired result.

□

THEOREM 3.26. *If $Bsw(G_r)$ is a biswapped network with basis network $r-regular$ graph G_r with order n, then*

$$\begin{aligned} KA^1_{a,b}(G_r) &= n^2 2^b (r+1)^{ab+1}, \\ RKA^1_{a,b}(G_r) &= n^2(r+1)2^b r^{ab}. \end{aligned}$$

Proof. The numbers of vertices and edges in $Bsw(G_r)$ are $2n^2$ and $n^2(r+1)$ respectively. By expressions in 1.1 and 1.2 we get the desired result. □

From Theorem 3.26, we obtain the following results.

COROLLARY 3.27. *If $Bsw(G_r)$ is a biswapped network with basis network $r-regular$ graph G_r with order n, then*

$$\begin{aligned} SO(G_r) &= n^2\sqrt{2}(r+1)^2, \\ {}^{m}SO(G_r) &= \frac{n^2}{\sqrt{2}}, \\ RSO(G_r) &= \sqrt{2}n^2(r+1)r, \\ {}^{m}RSO(G_r) &= \frac{n^2(r+1)}{r\sqrt{2}}, \\ RKA^2_{a,b}(G_r) &= n^2(r+1)r^{2ab}. \end{aligned}$$

DEFINITION 5. [9] *A walk of a graph G is an alternating sequence of points and lines $v_0, x_1, v_1, \ldots, v_{n-1}, x_n, v_n$, beginning and ending with points, in which each line is incident with the two points immediately preceding and following it. This walk joins v_0 and v_n, and may also be denoted $v_0\ v_1\ v_2 \ldots v_n$; it is sometimes called a $v_0 - v_n$ walk. It is closed if $v_0 = v_n$ and is open otherwise. It is a trail if all the lines are distinct, and a path if all the points(and thus necessarily all the lines) are distinct. If the walk is closed, then it is a cycle provided its n points are distinct and $n \geq 3$.*

COROLLARY 3.28. *If $Bsw(C_n)$ is a biswapped network with basis network $2-regular$ graph C_n with order n, then*

$$\begin{aligned} KA^1_{a,b}(C_n) &= n^2 2^b 3^{ab+1}, \\ RKA^1_{a,b}(C_n) &= 3n^2 2^{b(a+1)}, \\ SO(Bsw(C_n)) &= 9n^2\sqrt{2}, \\ {}^{m}SO(Bsw(C_n)) &= \frac{n^2}{\sqrt{2}}, \\ RSO(Bsw(C_n)) &= 6n^2\sqrt{2}, \end{aligned}$$

$$
\begin{aligned}
{}^{m}RSO(Bsw(C_n)) &= \frac{3n^2}{2\sqrt{2}}, \\
RKA^2_{a,b}(Bsw(C_n)) &= 3n^2 2^{2ab}.
\end{aligned}
$$

Proof. From Theorem 3.26 and Corollary 3.27, we obtain the desired result. □

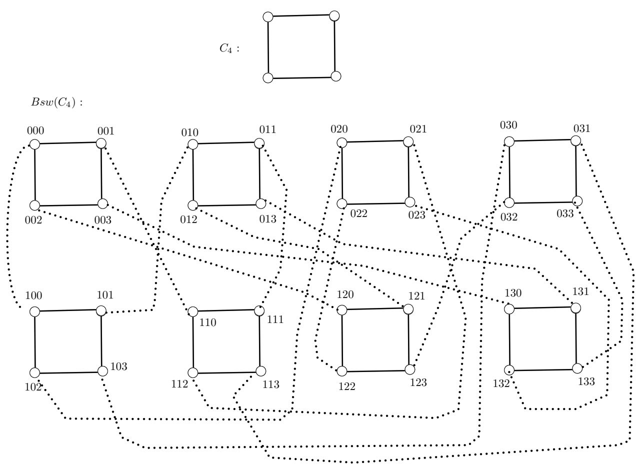


FIGURE 5. Biswapped network $Bsw(C_4)$

DEFINITION 6. [4] *A graph is complete if every two of its vertices are adjacent. A complete (n, m) graph is therefore a regular graph of degree $n-1$ having $m = \frac{n(n-1)}{2}$; we denote this graph by K_n.*

COROLLARY 3.29. *If $Bsw(K_n)$ is a biswapped network with basis network $(n-1)-$regular graph K_n with order n, then*

$$
\begin{aligned}
KA^1_{a,b}(K_n) &= n^3 2^b n^{ab}, \\
RKA^1_{a,b}(K_n) &= n^3 2^b (n-1)^{ab}, \\
SO(Bsw(K_n)) &= n^4\sqrt{2}, \\
{}^{m}SO(Bsw(K_n)) &= \frac{n^2}{\sqrt{2}}, \\
RSO(Bsw(K_n)) &= n^3(n-1)\sqrt{2}, \\
{}^{m}RSO(Bsw(K_n)) &= \frac{n^3}{\sqrt{2}(n-1)}, \\
RKA^2_{a,b}(Bsw(K_n)) &= n^3(n-1)^{2ab}.
\end{aligned}
$$

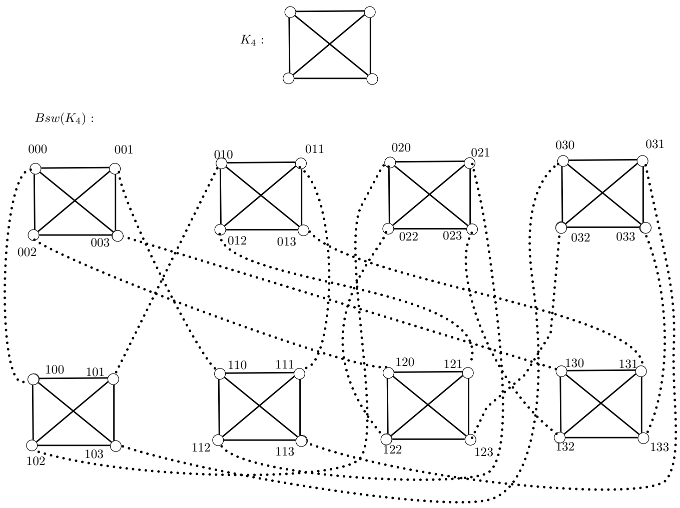


FIGURE 6. Biswapped network $Bsw(K_4)$

Proof. From Theorem 3.26 and Corollary 3.27, we obtain the desired result. □

DEFINITION 7. [9] *The n-cube Q_n is defined recursively by $Q_1 = K_2$ and $Q_n = K_2 \times Q_{n-1}$. Thus Q_n has 2^n points which may be labeled $a_1a_2 \dots a_n$, where each a_i is either 0 or 1. Two points of Q_n are adjacent if their binary representations differ at exactly one place.*

COROLLARY 3.30. *If $Bsw(Q_n)$ is a biswapped network with basis network $n-regular$ graph Q_n with order n, then*

$$\begin{aligned}
KA^1_{a,b}(Q_n) &= 2^{2n+b}(n+1)^{ab+1},\\
RKA^1_{a,b}(Q_n) &= 2^{2n+b}(n+1)n^{ab},\\
SO(Bsw(Q_n)) &= 2^{\frac{4n+1}{2}}\big(n+1\big)^2,\\
{}^mSO(Bsw(Q_n)) &= 2^{\frac{4n-1}{2}}\\
RSO(Bsw(Q_n)) &= 2^{\frac{4n+1}{2}}n\big(n+1\big),\\
{}^mRSO(Bsw(Q_n)) &= \frac{2^{\frac{4n-1}{2}}(n+1)}{n},\\
RKA^2_{a,b}(Bsw(Q_n)) &= 2^{2n}(n+1)n^{2ab}.
\end{aligned}$$

Proof. From Theorem 3.26 and Corollary 3.27, we obtain the desired result. □

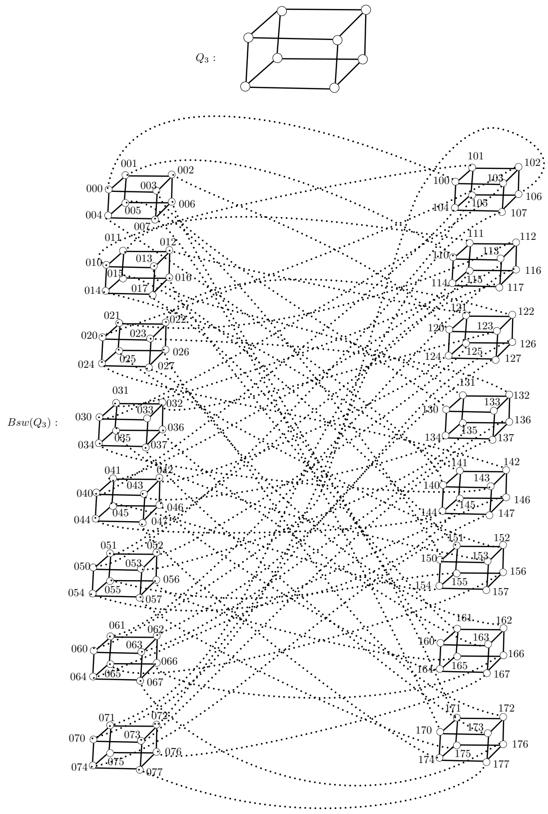


FIGURE 7. Biswapped network $Bsw(Q_3)$

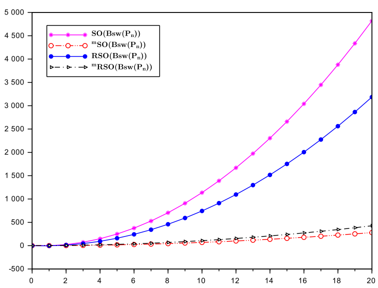


FIGURE 8. Graphical Comparison of $SO(Bsw(P_n))$, ${}^mSO(Bsw(P_n))$, $RSO(Bsw(P_n))$ and ${}^mRSO(Bsw(P_n))$

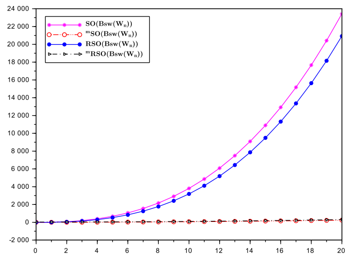
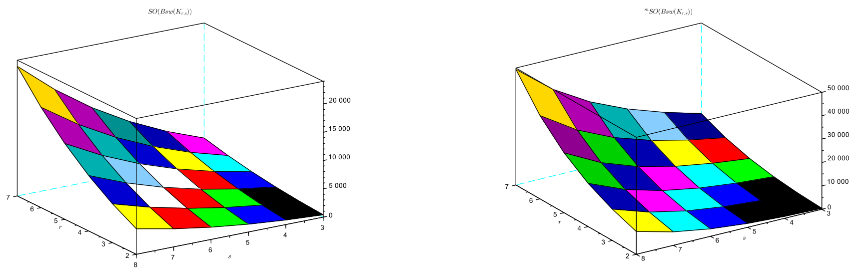
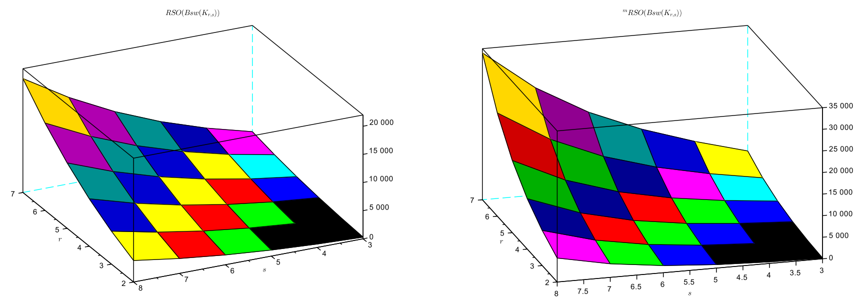


FIGURE 9. Graphical Comparison of $SO(Bsw(W_n)$, ${}^{m}SO(Bsw(W_n)$, $RSO(Bsw(W_n)$ and ${}^{m}RSO(Bsw(W_n)$.



(A) $SO(Bsw(K_{r,s})$ and ${}^{m}SO(Bsw(K_{r,s})$



(B) $RSO(Bsw(K_{r,s})$ and ${}^{m}RSO(Bsw(K_{r,s})$

FIGURE 10. Graphical Comparison of $SO(Bsw(K_{r,s}))$, ${}^{m}SO(Bsw(K_{r,s})$, $RSO(Bsw(K_{r,s})$ and ${}^{m}RSO(Bsw(K_{r,s})$.

4. Graphical representation and discussion

In this section we discuss graphically representation related to the sombor index, modified sombor index, reduced sombor index, reduced, modified sombor index. All these indices are degree based topological indices for the OTIS biswapped network of the basis graph as path, wheel and complete bipartite graphs.
We determined the explicit formulas for the first (a, b)-KA index, the Sombor index, the modified Sombor index, the first reduced (a, b)-KA index, the reduced Sombor index, the reduced modified Sombor index, the second reduced (a, b)-KA index and the mean Sombor index mSO_α for the OTIS biswapped networks.

- The graphical representation of OTIS biswapped network of path graph $Bsw(P_n)$ is shown in Figure 8. It can be observed that the values of all indices increase with increasing value of n.
- Similarly, the graphical representation of OTIS biswapped network of wheel graph $Bsw(W_n)$ is shown in Figure 9. It can be observed that the values of all indices increase with increasing value of n.
- Similarly, the graphical representation of OTIS biswapped network of complete bipartite graph $Bsw(K_{r,s})$ is shown in Figure 10. It can be observed that the values of all indices increase with increasing values of r, s.

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